basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

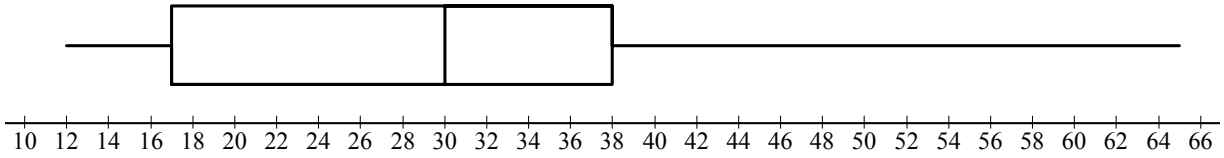
MARKS/*PUNTE*: 150

These marking guidelines consist of 19 pages.
Hierdie nasienriglyne bestaan uit 19 bladsye.

NOTE:
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION/*VRAAG* 1



1.1.1	$\min = 12$ $Q_1 = 17$ median / *mediaan* $= 30$ $Q_3 = 38$ $\max = 65$	✓ min + max ✓ median, Q_1 and/en Q_3 (2)
1.1.2	$IQR = Q_3 - Q_1$ $= 38 - 17$ $= 21$	✓ answer/*antw* (1)
1.1.3	Skewed to the right OR positively skewed *Skeef na regs* *OF positief skeef*	✓ answer/*antw* (1)

5	8	10	17	20	29	32	48	50	50	63	y	107

1.2.1	Mean/*Gemiddeld* $= \frac{439 + y}{13}$ $41 = \frac{439 + y}{13}$ $439 + y = 533$ $y = 94$	✓ $41 = \frac{439 + y}{13}$ ✓ answer/*antw* (2)
1.2.2	$\sigma = 30{,}94$	✓ answer/*antw* (1)

1.2.3	$41\times13=533$ $6\times18=108$ $\frac{533+108}{19}=\frac{641}{19}=33{,}74$	✓108 ✓ $533+108=641$ ✓ answer/*antw* (3) **[10]**

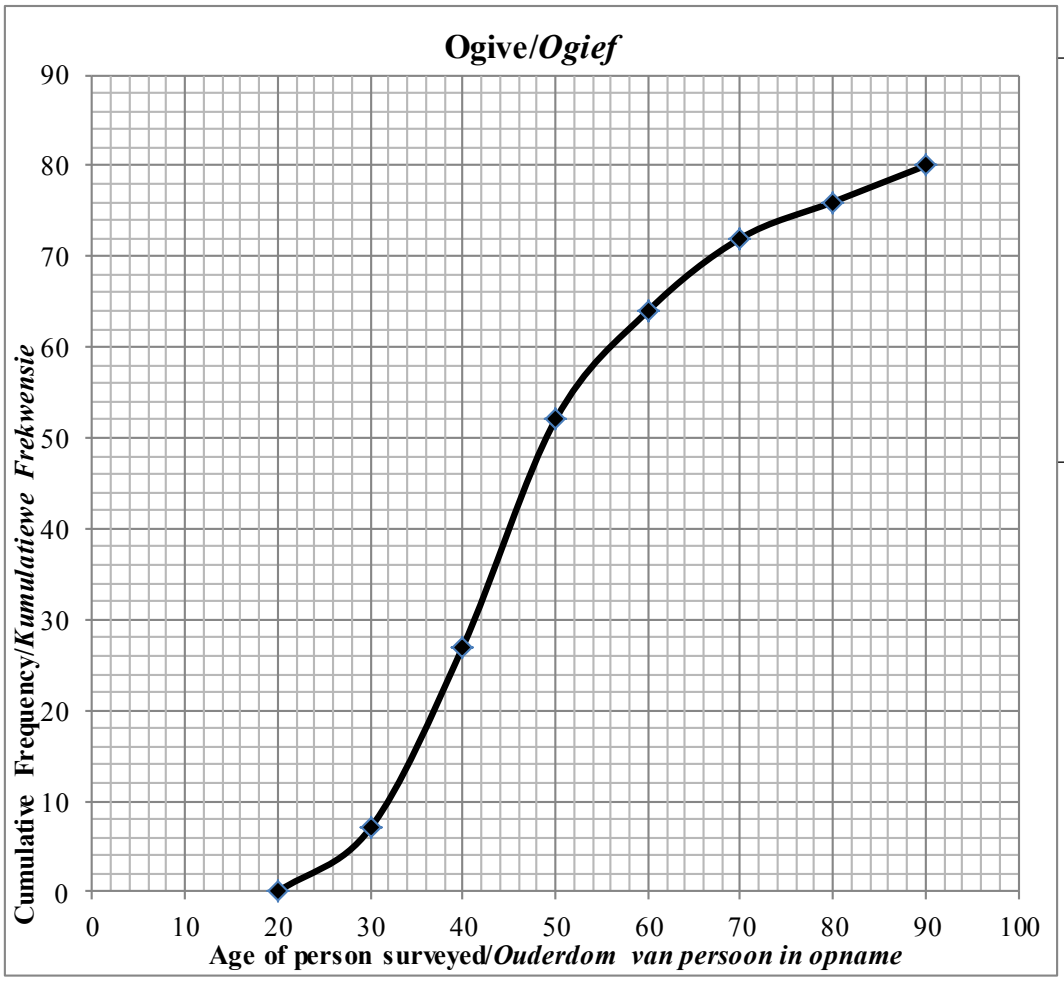
QUESTION/*VRAAG* 2

2.1		✓ 20, 12 ✓ 8, 4 ✓ 52 ✓76 (4)

AGE OF PERSON SURVEYED/*OUDERDOM VAN PERSOON IN OPNAME*	**FREQUENCY/*FREKWENSIE***	**CUMULATIVE FREQUENCY/ *KUMULATIEWE FREKWENSIE***
$20 < x \leq 30$	7	7
$30 < x \leq 40$	**20**	27
$40 < x \leq 50$	25	**52**
$50 < x \leq 60$	**12**	64
$60 < x \leq 70$	**8**	72
$70 < x \leq 80$	4	**76**
$80 < x \leq 90$	**4**	80

2.2	$n = 80$	✓answ/*antw* (1)
2.3	$40 < x \leq 50$	✓ answ/*antw* (1)

2.4



✓Grounding (20; 0) /*Geanker by (20; 0)*

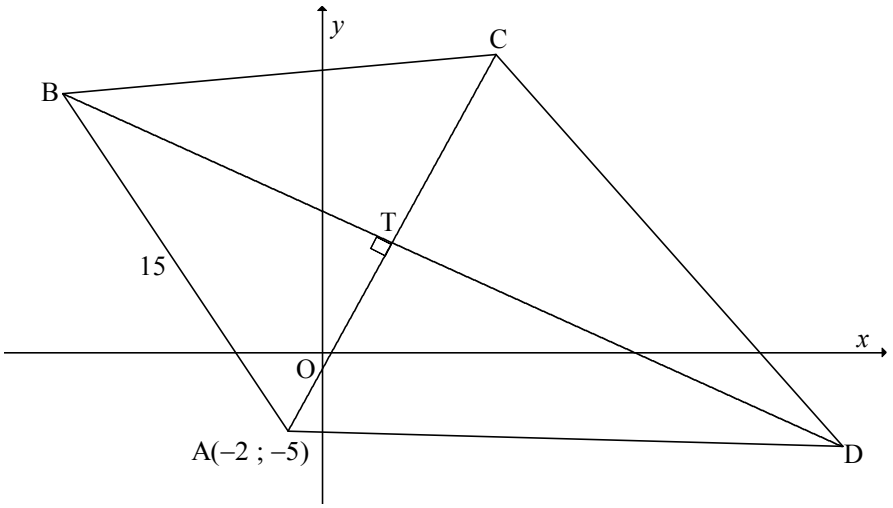
✓upper limits/ *boonste limiete*

✓ shape (smooth curve)/ *vorm (gladde kurwe)*

(3)

2.5	$80 - 58 = 22$ $\frac{22}{80} \times 100 = 27{,}5\%$	**Accept/*aanvaar*:** 56 – 59 calls/*oproepe*	✓58 calls/*oproepe* ✓22 ✓ 27,5% (3)
			[12]

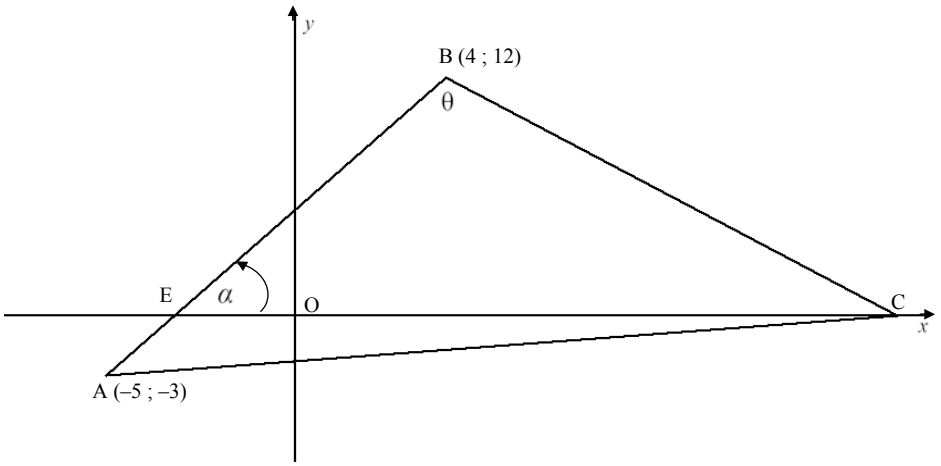
QUESTION/*VRAAG* 3



3.1	$BD \quad y=-\frac{1}{2}x+9$ $\therefore m_{BD}=-\frac{1}{2}$ $\therefore m_{AC}=2$	✓ Standard form/*vorm* ✓ answ/*antw* (2)
3.2	$y-y_1=m(x-x_1)$ $y-(-5)=2(x-(-2))$ $y=2x-1$	✓ subst (–2 ; –5) ✓ answ/*antw* (2)
3.3	$2x-1=-\frac{1}{2}x+9$ $\frac{5}{2}x=10$ $x=4$ $y=2(4)-1$ $y=7$ $\mathrm{T}(4\ ;\ 7)$	✓ $2x-1=-\frac{1}{2}x+9$ ✓ $x=4$ ✓ $y=7$ (3)

3.4.1	$4=\frac{-2+x}{2}$ $8=-2+x$ $x=10$ $7=\frac{-5+y}{2}$ $14=-5+y$ $y=19$ $C(10\,;19)$	✓ $x=10$ ✓ $y=19$ (2)
3.4.2	$AT=\sqrt{(4-(-2))^2+(7-(-5))^2}$ $=\sqrt{180}$ $=6\sqrt{5}$ $BT^2+AT^2=AB^2$ $(Pythagoras)$ $BT=\sqrt{15^2-\left(\sqrt{180}\right)^2}$ $=3\sqrt{5}$	✓ subst. in distance/*afstand* form. ✓ answer/*antw* ✓ subst. in pyth ✓ answer/*antw* (4)
3.4.3	BC is the diameter/ *middellyn* [subt. right / *ondersp. reg* $\angle$] or/o $[\text{conv.}\angle^s \text{ in semi - circle/ } omgk.\ \angle^s\ in\ halfsirkel]$ Radius $=\frac{15}{2}=7{,}5$ units/ *eenh.*	✓ S ✓ answ/*antw* (2) **[15]**

QUESTION/*VRAAG* 4

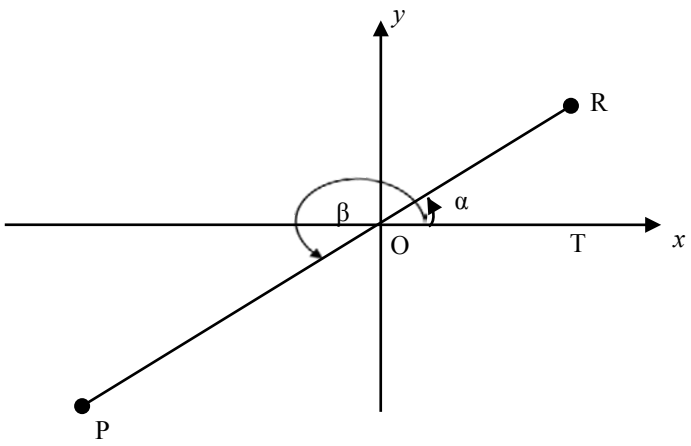


4.1	$m_{AB}=\frac{12-(-3)}{4-(-5)}=\frac{5}{3}$ **OR/*OF*** $m_{AB}=\frac{-3-12}{-5-4}=\frac{5}{3}$	✓ subst. in gradient form. ✓ answ/*antw* (2)
4.2	$y-12=\frac{5}{3}(x-4)$ $0-12=\frac{5}{3}(x-4)$ $x=-\frac{16}{5}$ $\text{E}\left(-\frac{16}{5}\ ;\ 0\right)$ **OR/*OF*** $\frac{0-12}{x-4}=\frac{5}{3}$ $\qquad$ $\frac{0+3}{x+5}=\frac{5}{3}$ $-36=5x-20$ $\qquad$ $9=5x+25$ $-16=5x$ $\qquad$ $-16=5x$ $x=\frac{-16}{5}$ $\qquad$ $x=\frac{-16}{5}$ $\text{E}\left(-\frac{16}{5}\ ;\ 0\right)$	✓equation/*verg.* ✓ $y=0$ ✓ answ/*antw* (3) ✓equating/*verg.* ✓ $y=0$ ✓ answ/*antw* (3)

4.3	$\tan\alpha = m_{AB}$ $\tan\alpha = \frac{5}{3}$ $\alpha = 59^o$	✓ $\tan\alpha = \frac{5}{3}$ ✓ $\alpha = 59^o$ (2)
4.4	$76° + 59° = 135°$ [ext $\angle$ of Δ] $B\hat{C}X = 135°$ $\tan 135° = m_{BC}$ $m_{BC} = -1 = m_{//}$ $y - (-3) = -1(x - (-5))$ $y = -x - 8$	✓ 135° ✓ $\tan 135^o = m_{BC}$ ✓ answer/*antw* ✓ subst (–3 ; –5) ✓ answer/*antw* (5) [**12**]

QUESTION/*VRAAG* 5

5.1	$\sin(90^o - x).\cos(180^o + x) + \tan x.\cos x.\sin(x - 180^o)$ $= \cos x.(-\cos x) + \frac{\sin x}{\cos x}.\cos x.(-\sin x)$ $= -\cos^2 x - \sin^2 x$ $= -(\cos^2 x + \sin^2 x)$ $= -1$	✓ $\cos x$ ✓ $-\sin x$ ✓ $-\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓common factor/*gemene fakt.* ✓ identity/*identiteit* (6)
5.2	$\text{LHS} = \frac{\sin 315°.\tan 210^o.\sin 190°}{\cos 100°.\sin 120^o}$ $= \frac{(-\sin 45°).(\tan 30^o).(-\sin 10°)}{(-\sin 10°).(\sin 60^o)}$ $= \frac{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}}$ $= -\frac{\sqrt{2}}{3}$	✓ $-\sin 45°$ ✓ $\tan 30°$ ✓ $-\sin 10°$ ✓ $-\sin 10°$ ✓ $\sin 60°$ ✓ subst. of special angles/*inverv. van sp hoeke* (6)

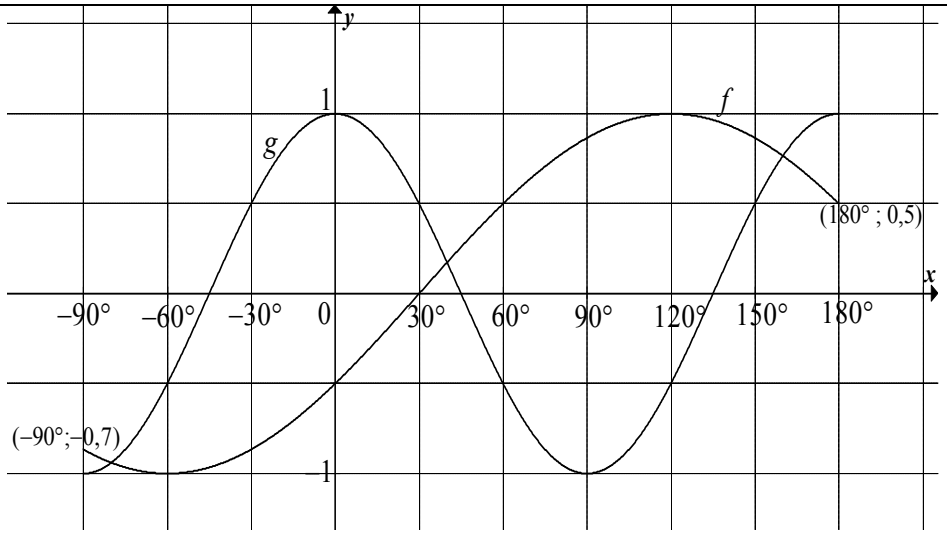


5.3.1	$x^2 + y^2 = r^2$ [Pythagoras] $(x)^2 + (3)^2 = 5^2$ $x^2 = 16$ $x = 4$ $R(4\ ;\ 3)$ $\tan\alpha = \frac{3}{4}$	✓ subst in pyth ✓ $x = 4$ ✓ answer/*antw* (3)
5.3.2	$\sin\beta$ $= \sin(180^\circ + \alpha)$ $= -\sin\alpha$ $= \frac{-3}{5}$	✓ $\beta = 180° + \alpha$ ✓ $-\sin\alpha$ ✓ answer/*antw* (3)

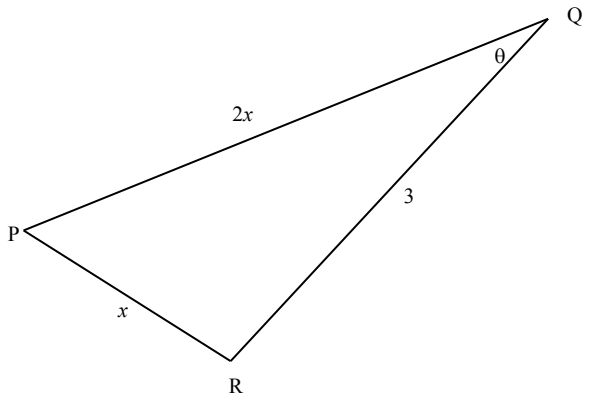
5.3.3	$\frac{y}{10}=\frac{-3}{5}$ $y=-6$ $\therefore x=-8$ $P(-8;-6)$	✓ $\frac{y}{10}$ ✓ $\frac{-3}{5}$ ✓ $y=-6$ ✓ $x=-8$ (4)
5.4.	$\text{LHS}=\frac{\sin\theta-\tan\theta.\cos^2\theta}{\cos\theta-(1-\sin^2\theta)}$ $=\frac{\sin\theta-\frac{\sin\theta}{\cos\theta}.\cos^2\theta}{\cos\theta-\cos^2\theta}$ $=\frac{\sin\theta(1-\cos\theta)}{\cos\theta(1-\cos\theta)}$ $=\tan\theta$ $=\text{RHS}$ **OR/*OF*** $\text{LHS}=\frac{\sin\theta-\tan\theta.\cos^2\theta}{\cos\theta-1+(1-\cos^2\theta)}$ $=\frac{\sin\theta-\frac{\sin\theta}{\cos\theta}.\cos^2\theta}{\cos\theta-\cos^2\theta}$ $=\frac{\sin\theta(1-\cos\theta)}{\cos\theta(1-\cos\theta)}$ $=\tan\theta$ $=\text{RHS}$	✓ $\frac{\sin\theta}{\cos\theta}$ ✓ $\cos^2\theta$ ✓ common fact/ *gemene fakt.* ✓ common fact/ *gemene fakt.* (4) ✓ $\frac{\sin\theta}{\cos\theta}$ ✓ $1-\cos^2\theta$ ✓ common fact/ *gemene fakt.* ✓ common fact/ *gemene fakt.* (4) **[26]**

QUESTION/*VRAAG* 6

6.1	$\sin(x-30^\circ)=\cos 2x$ $\sin(x-30^\circ)=\sin(90^\circ-2x)$ $x-30^\circ=90^\circ-2x+360^\circ k$ *or* $x-30^\circ=180^\circ-(90^\circ-2x)+360k$ $3x=120^\circ+360^\circ k$ $\quad -x=120^\circ+360^\circ k$ $x=40^\circ+120^\circ k$ $\quad x=-120^\circ+360^\circ k, k\in Z$ **OR/*OF*** $\cos(90^\circ-(x-30^\circ))=\cos 2x$ $\cos(120^\circ-x)=\cos 2x$ $120^\circ-x=2x+360^\circ k$ *or* $120^\circ-x=-2x+360^\circ k$ $-3x=-120^\circ+360^\circ k$ $\quad x=-120^\circ+360^\circ k$ $x=40^\circ+120^\circ k,\ k\in Z$ **NOTE/LET WEL:** $x=-120^\circ+k.360^\circ$ is equivalent to/*ekwivalent aan* $x=240^\circ+k.360^\circ$	✓ $\sin(90^\circ-2x)$ ✓ $x-30^\circ=90^\circ-2x+360^\circ k$ ✓ $x=40^\circ+120^\circ k$ ✓ $x-30^\circ=180^\circ-(90^\circ-2x)+360k$ ✓ $x=-120^\circ+360^\circ k$ (5) ✓ $\cos(90^\circ-(x-30^\circ))$ ✓ $120^\circ-x=2x+360^\circ k$ ✓ $x=40^\circ+120^\circ k$ ✓ $120^\circ-x=-2x+360^\circ k$ ✓ $x=240^\circ+360^\circ k$ (5)
6.2.1	180°	✓ answer/*antw* (1)
6.2.2	$-1\le y\le 1$ **OR/*OF*** $y\in[-1;1]$	✓ values/*waardes* ✓ notation/*notasie* (2) ✓ values/*waardes* ✓ notation/*notasie* (2)

6.2.3		✓ *f* *x*-intercept at/*afsnit by* 30° ✓ shape of/ *vorm van f* ✓ *g* *x*-intercepts at/*afsnitte by* 45° and/ *en* 135° ✓ shape of/*vorm van g* ✓TP at/*DP by* $(90^\circ;-1)$ (5)
6.2.4	$x=-80^\circ\ ;\ x=40^\circ\ ;\ x=160^\circ$	✓✓✓one mark per answer/*een punt per antw.*(3)
		[16]

QUESTION/*VRAAG* 7



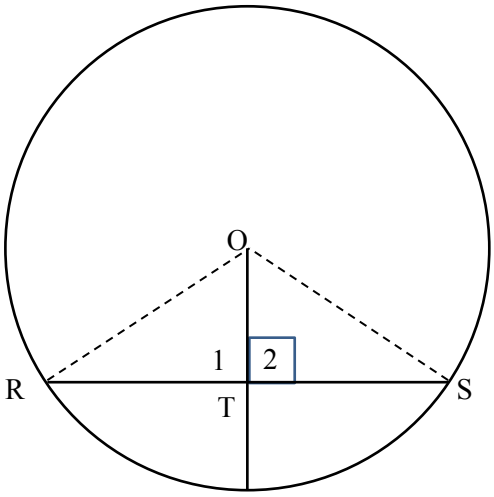
7.1	$x^2=(2x)^2+(3)^2-2(2x)(3)\cos\theta$ $12x\cos\theta=3x^2+9$ $\cos\theta=\dfrac{3x^2+9}{12x}$ $\cos\theta=\dfrac{3(x^2+3)}{12x}$ $\cos\theta=\dfrac{x^2+3}{4x}$	✓ cos rule ✓ subst ✓ simplify/*vereenv* (3)

7.2.1	$\cos\theta = \dfrac{(2,4)^2 + 3}{4(2,4)}$ $\cos\theta = 0,9125$ $\theta = 24,15^{o}$	✓ subst ✓ $\cos\theta = 0,9125$ ✓ answer/*antw.* (3)
7.2.2	Area of/*van* $\Delta PQR = \dfrac{1}{2} \times PQ \times QR \times \sin\hat{Q}$ $= \dfrac{1}{2} \times 4,8 \times 3 \times \sin 24,15$ $= 2,95$ units/*eenh*^{2}	✓ subst ✓ answer/*antw.* (2)
7.3	For/*vir* $x > 0$, $\cos\theta > 0$ $0° < \theta < 90°$ $0 < \dfrac{x^2 + 3}{4x} < 1$ $x^2 + 3x < 4x$ $x^2 - 4x + 3 < 0$ $(x-1)(x-3) < 0$ $1 < x < 3$	✓✓ $0 < \dfrac{x^2 + 3}{4x} < 1$ ✓✓ $1 < x < 3$ (4)
		[12]

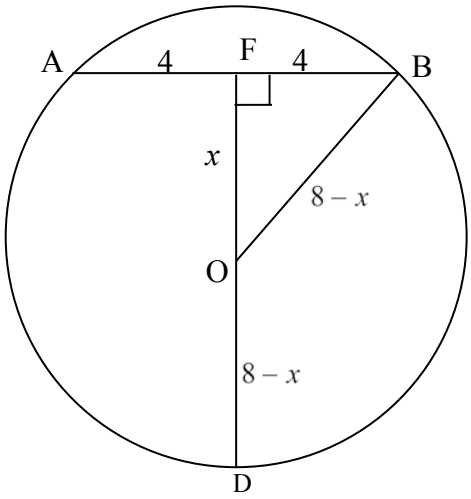
QUESTION 8/*VRAAG* 8

8.1	$V = \frac{1}{3}Ah$ $640 = \frac{1}{3} \times (16 \times 16) \times h$ $h = 7{,}5cm$	✓ Area of square/*van vierk.* $= (16 \times 16)$ ✓ Subst in volume form (2)
8.2	slant height / *skuinshoogte* $= \sqrt{7{,}5^2 + 8^2} = 10{,}9658....$ Total surface / *Totale buite area* $= (side \times side) + 4\left(\frac{1}{2}b \times \perp h\right)$ $= (16 \times 16) + 4\left(\frac{1}{2} \times 16 \times 10{,}9658....\right)$ $= 606{,}91 \text{cm}^2$	✓Subst in pyth ✓answer/*antw* ✓Subst in SA/ *BO* form. ✓answer/*antw* (4) **[6]**

QUESTION 9/*VRAAG* 9

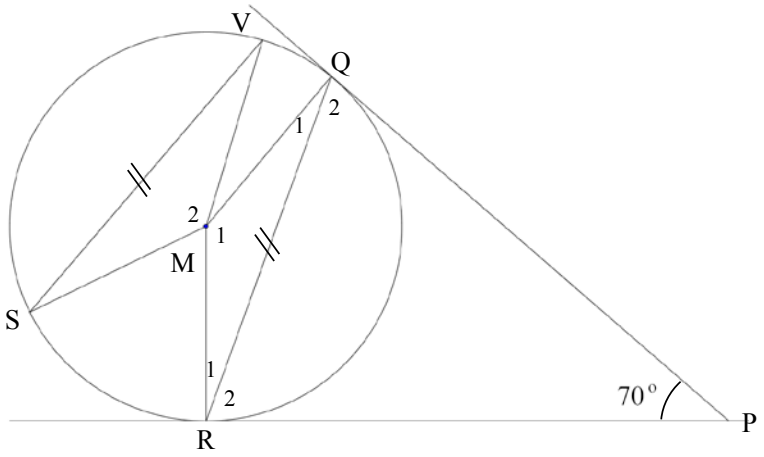


9.1	Construction/*Konstr*: Draw/*trek* radii OR and/*en* OS In ΔOTR and/ *en* ΔOTS $OR = OS$ (radii) $OT = OT$ (common side/ *gemene sy*) $\hat{T}_1 = \hat{T}_2 = 90^o$ ($\angle^s$ on straight line/ *op 'n reguit lyn*) $\Delta OTR \equiv \Delta OTS$ (90^o HS) $\therefore\ RT = TS$ ($\equiv \Delta^s$)	✓Constr/*Konstr* ✓S (OT is common/*gemeen*) ✓S/R ✓R ✓S (5)



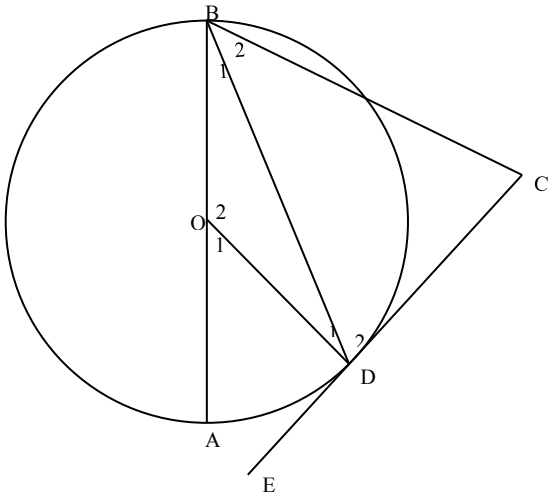
9.2	$AF = FB = 4cm$ $\begin{bmatrix} \text{line from centre} \perp \text{to chord/} \\ lyn\ van\ mdpt \perp aan\ koord \end{bmatrix}$ $\text{OD} = OB = 8 - x (radii)$ $OB^2 = OF^2 + FB^2\ (Pythagoras)$ $(8-x)^2 = x^2 + 4^2$ $64 - 16x + x^2 = x^2 + 4^2$ $48 = 16x$ $x = 3$ length of/ *lengte van radius* $= 8 - x$ $= 8 - 5$ $= 3\ units / eenh$	✓S/R ✓$8-x$ ✓$(8-x)^2 = x^2 + 4^2$ ✓$x = 3$ ✓Answer/*antw* (5) **[10]**

QUESTION/*VRAAG* 10



10.1	$\hat{Q}_2 = \hat{R}_2$ [tangents from common point/ *rk ln e van selfde punt*] $\hat{Q}_2 + \hat{R}_2 + 70^o = 180^o$ [sum $\angle\Delta$] $2\hat{R}_2 = 110^o$ $\hat{R}_2 = 55^o$	✓ S ✓R ✓ S ✓ $\hat{R}_2 = 55^o$ (4)
10.2	$\hat{Q}_2 + \hat{Q}_1 = 90^o$ [tan/*rkl* $\perp$ rad] $\hat{Q}_1 = 35^o$ **OR/*OF*** $\hat{R}_1 + \hat{R}_2 = 90^o$ [tan/*rkl* $\perp$ rad] $\hat{R}_1 = 35^o$ $\hat{Q}_1 = \hat{R}_1 == 35^o$ [$OR = OQ$]	✓ R ✓ $\hat{Q}_1 = 35^o$ (2) ✓ R ✓ $\hat{Q}_1 = 35^o$ (2)
10.3	$\hat{O}_1 + \hat{R}_1 + \hat{Q}_1 = 180^o$ [sum $\angle\Delta$] $\hat{O}_1 = 180^o - 70^o = 110^o$ $\hat{O}_2 = 110^o$ [equal chords subtend = $\angle$ at the centre/ *gelyke koorde onrsp.* = $\angle$ *by mdpt*]	✓ $\hat{O}_1 = 110^o$ ✓ $\hat{O}_2 = 110^o$ ✓ R (3) **[9]**

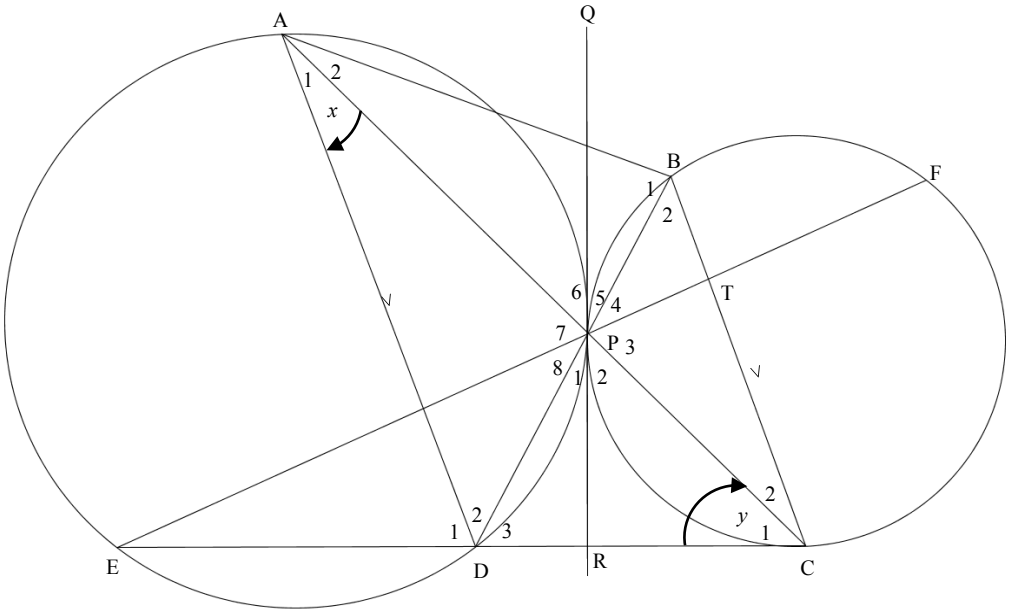
QUESTION/*VRAAG* 11



11.1	$\hat{B}_1 = \hat{B}_2 = x$ [BD bisect/*halveer* $\angle$ A$\hat{B}$C] $A\hat{B}C = 2x$ $\hat{O}_1 = 2x$ [$\angle$ at centre = 2 times $\angle$ at circumference/] [*midpts*$\angle$ = *2* × *omtreks*$\angle$] $\therefore$ BC $\|\|$ OD [corresponding $\angle$ are equal/*ooreenk.* $\angle$ *is gelyk*] **OR/*OF*** $\hat{B}_1 = \hat{B}_2 = x$ [BD bisect/*halveer* $\angle$ A$\hat{B}$C] $\hat{D}_1 = x$ [angle opp = sides/$\angle$*e to gelyke sye*] $\hat{D}_1 = \hat{B}_2 = x$ $\therefore$ BC $\|\|$ OD [alternate angles are equal/*verw* $\angle$*e gelyk*] **OR/*OF*** $\hat{B}_1 = \hat{B}_2 = x$ [BD bisect/*halveer* $\angle$ A$\hat{B}$C] $A\hat{B}C = 2x$ $\hat{O}_1 = 2x$ [angle at centre = 2 times angle at circumference] [*midpts*$\angle$ = *2* × *omtreks*$\angle$] $\hat{O}_2 = 180^\circ - 2x$ [$\angle$ on a straight line/$\angle$*op reguit lyn*] $\hat{O}_2 + A\hat{B}C = 180^\circ - 2x + 2x = 180^\circ$ $\therefore$ BC $\|\|$ OD [co-int angles are suppl/*ko-binne* $\angle$ *is suppl*]	✓ S ✓ S ✓ R ✓ R ✓ S ✓ S ✓ R ✓ R ✓ S ✓ S ✓ R ✓ R 	(4) (4) (4)

11.2	$O\hat{D}C = 90^o$ [tan/*rkl* $\perp$ rad] $\hat{C} = 90^o$ [co – int / *ko-binne* $\angle$'s OD$\|$BC] **OR/*OF*** $\hat{D}_1 = x$ $\hat{D}_2 = 90^o - x$ [tan/*rkl* $\perp$ rad] $\hat{C} = 180^o - (90^o - x) - x$ [int $\angle$'s of / *van* Δ] $= 90^o$ **OR/*OF*** $E\hat{D}C = 90^o$ [tan/*rkl* $\perp$ rad] $\hat{C} = 90^o$ [*corresp.* / *ooreenk.* $\angle$'s OD$\|$BC]	✓ S/R ✓ S ✓R (3) ✓ S/R ✓ S ✓R (3) ✓ S/R ✓ S ✓R (3) **[7]**

QUESTION/*VRAAG* 12



12.1	$\hat{P}_1 = x$ [tan – ch th/ *rkl-kdst*] $\hat{C}_2 = x$ [alt / *verw.*$\angle^s$ AD $\parallel$ *B*C] $\hat{E} = x$ [$\angle^s$ in the same segment/*dieselfde segment*] $\hat{P}_5 = x$ [vert opp/ *reg oorst*]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R ✓ S/R (7)
12.2	$D\hat{C}B = x + y$ $E\hat{D}A = \hat{D}_1 = x + y$ [corresp / *ooreenk.* $\angle^s$ = , AD $\parallel$ BC] $\therefore E\hat{P}A = x + y$ [$\angle^s$ in the same segment/*dieselfde segment*]	✓S✓ R ✓S ✓R (4)
12.3	$\hat{P}_2 = y$ [tan from a commom point/ *rklyne v dieselfde pt*] $D\hat{P}T = \hat{P}_1 + \hat{P}_2 + \hat{P}_3$ $= x + y + (x + y)$ $= 2x + 2y$ $\hat{C} = x + y$ $D\hat{P}T + \hat{C} = 180^o$ [opp $\angle^s$ of a cyclic quad/ *teenoorst.* $\angle^s$ *van kvhk]* $2x + 2y + x + y = 180^o$ $3x + 3y = 180^o$ $\therefore x + y = 60^o$	✓ S/R ✓ $\hat{C} = x + y$ ✓ Subst ✓ Answ/*antw* (4) **[15]**

TOTAL/*TOTAAL*: 150